

HD-MPC Four-Tank Plant Benchmark

Application and evaluation of distributed predictive controllers

October 5, 2010

1 Four-tanks plant model

The four tanks plant is a multivariable laboratory plant of interconnected tanks with nonlinear dynamics and subject to state and input constraints. A scheme of this plant is presented in Figure 1. The real experimental plant is placed at the University of Seville control laboratory.

A state space continuous time model of the quadruple tank process system can be derived from first principles as follows

$$\begin{aligned}\frac{dh_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_a}{A_1}q_a \\ \frac{dh_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_b}{A_2}q_b \\ \frac{dh_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_b)}{A_3}q_b \\ \frac{dh_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_a)}{A_4}q_a\end{aligned}\tag{1}$$

The plant parameters, estimated on the real plant are shown in the following table:

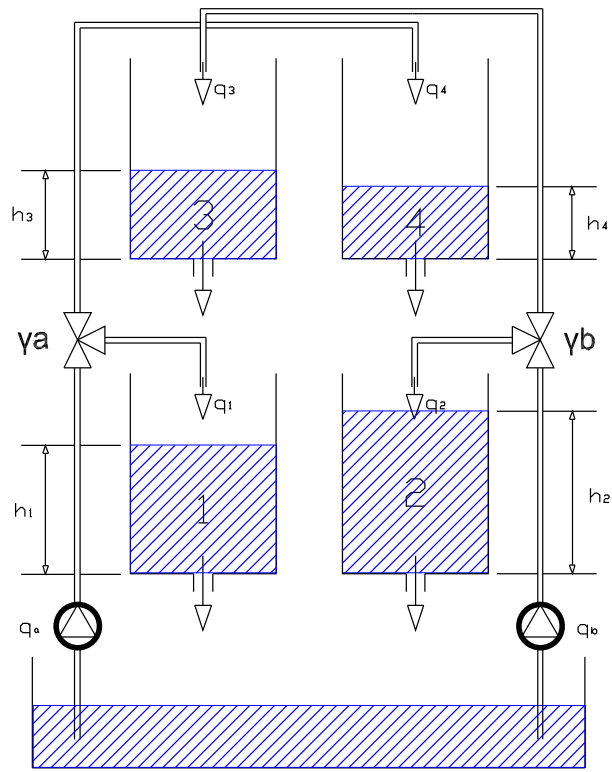


Figure 1: 4 Plant Schema

	Value	Unit	Description
H_{1max}	1.36	m	Maximum level of the tank 1
H_{2max}	1.36	m	Maximum level of the tank 2
H_{3max}	1.30	m	Maximum level of the tank 3
H_{4max}	1.30	m	Maximum level of the tank 4
H_{min}	0.2	m	Minimum level in all cases
Q_{amax}	3.26	m^3/h	Maximal flow of pump A
Q_{bmax}	4	m^3/h	Maximal flow of pump B
Q_{min}	0	m^3/h	Minimal flow
Q_a^0	1.63	m^3/h	Equilibrium flow
Q_b^0	2.0000	m^3/h	Equilibrium flow
a_1	1.310e-4	m^2	Discharge constant of tank 1
a_2	1.507e-4	m^2	Discharge constant of tank 2
a_3	9.267e-5	m^2	Discharge constant of tank 3
a_4	8.816e-5	m^2	Discharge constant of tank 4
A	0.06	m^2	Cross-section of all tanks
γ_a	0.3		Parameter of the 3-ways valve
γ_b	0.4		Parameter of the 3-ways valve
h_1^0	0.6534	m	Equilibrium level of tank 1
h_2^0	0.6521	m	Equilibrium level of tank 2
h_3^0	0.6594	m	Equilibrium level of tank 3
h_4^0	0.6587	m	Equilibrium level of tank 4
T_m	5	s	Sample time

The minimum level of the tanks has been taken greater than zero to prevent eddy effects in the discharge of the tank. One important property of this plant is that the dynamics present multivariable transmission zeros which can be located in the right hand side of the s plane for some operating conditions. Hence, the values of γ_a and γ_b have been chosen in order to obtain a system with non-minimum phase multivariable zeros.

Linearizing the model at an operating point given by h_i^0 and defining the deviation variables $x_i = h_i - h_i^0$ and $u_j = q_j - q_j^0$ where $j = a, b$ and $i = 1, \dots, 4$ we have that:

$$\begin{aligned} \frac{dx}{dt} &= \begin{bmatrix} \frac{-1}{\tau_1} & 0 & \frac{A_3}{A_1\tau_3} & 0 \\ 0 & \frac{-1}{\tau_2} & 0 & \frac{A_4}{A_2\tau_4} \\ 0 & 0 & \frac{-1}{\tau_3} & 0 \\ 0 & 0 & 0 & \frac{-1}{\tau_4} \end{bmatrix} x + \begin{bmatrix} \frac{\gamma_a}{A_1} & 0 \\ 0 & \frac{\gamma_b}{A_2} \\ 0 & \frac{(1-\gamma_b)}{A_3} \\ \frac{(1-\gamma_a)}{A_4} & 0 \end{bmatrix} u. \\ y &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x \end{aligned}$$

where $\tau_i = \frac{A_i}{a_i} \sqrt{\frac{2h_i^0}{g}} \geq 0$, $i = 1, \dots, 4$, are the time constants of each tank. This model has been discretized using the zero-order hold method with a sampling time of 5 seconds.

A detailed description of the plant can be found in the attached documents.

2 Control Problem

The objective of the benchmark is to test and compare centralized, decentralized, and distributed predictive controllers under similar operation conditions. To this end the following experiment is defined in which the controllers must regulate the levels of tanks 1 and 2 to follow a set of reference changes by manipulating the inlet flows q_a and q_b based on the measured levels of the four tanks:

- The first set-points are set to $s_1 = 0.65$ m and $s_2 = 0.65$ m. This first reference is aimed to steer the plant to the operation point. Once the plant is in the operation point the test begins maintaining the operation point during 300 seconds.
- In the first step, the reference is changed to $s_1 = 0.3$ m and $s_2 = 0.3$ m during 3000 seconds.
- Then, the reference is changed to $s_1 = 0.5$ m and $s_2 = 0.75$ m during 3000 seconds.

- Finally, the set-points are changed to $s_1 = 0.9$ m and $s_2 = 0.75$ m during 3000 seconds. To perform this change tanks 3 and 4 have to be emptied and filled respectively.

The set-point signals are shown in Figure 2. The total control test takes 9300 seconds.

The objective of the benchmark is to design the distributed MPC controllers to optimize the following performance index:

$$J = \sum_{i=0}^{N_{sim}} (h_1(i) - s_1(i))^2 + (h_2(i) - s_2(i))^2 + 0.01(q_a(i) - q_a^s(i))^2 + 0.01(q_b(i) - q_b^s(i))^2$$

where q_a^s and q_b^s are the steady manipulable variables of the plant for the set-points s_1 and s_2 calculated from steady conditions of the proposed model of the plant. The proposed sampling times is 5 seconds, that is, $N_{sim} = 1860$ samples.

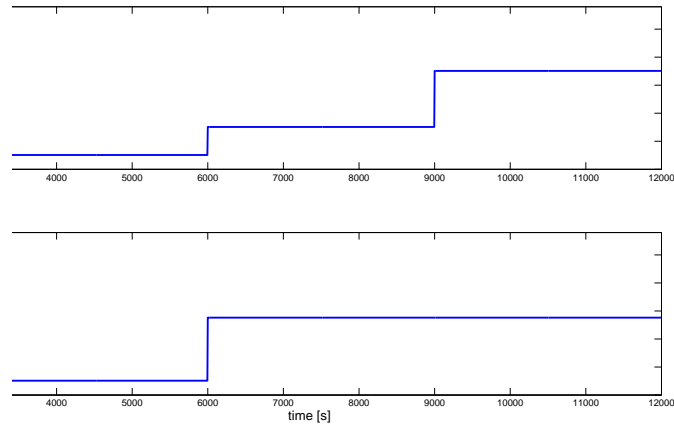


Figure 2: Set-point signals for the benchmark

The evaluation and comparison between the different controllers will be performed according to a collection of indexes. These are aimed to compare different properties for the controllers as well as their behavior in the control test. These indexes are the following:

- Evaluation of the controller (Qualitative Indexes)
 1. Modelling requirements: the class of model considered by each of the controllers, for instance linear/nonlinear, plant model or subsystem model, etc.
 2. Controller objectives: the properties addressed by the tested controllers, for instance optimality, constraint satisfaction, stabilizing design, recursive feasibility, etc.

3. Auxiliary software needed: optimization routines, simulation routines, etc.
- Evaluation of the test (Quantitative)
 1. Performance index J : gives a measure of the performance of the whole trajectory of the controlled plant.
 2. Performance index during the transient J_t : gives a measure of the performance index measured during the transients of the trajectory. This allows to remove the effect of the steady offset.
 3. Settling time: gives a measure of the velocity of the controlled plant. This is calculated by summing all the settling times (for 95%) of the steps in the reference.
 4. The number of floating point numbers in the data packet transmitted by the controllers: the total number of floating point numbers sent by one controller to the other during a sampling time.
 5. Number of data packets transmitted during a sampling time: number of times that each controller sends data to the other controller.

Every controller will be designed using the same simulation model implemented in MATLAB/Simulink. Each designed controller will be implemented as a Simulink block.

3 Data and programs provided

The accuracy of the model of the plant allows us to obtain reliable controllers designed on this model. To this aim a Simulink model for simulation is provided. This model is represented in figure 3.

This model contains a block *Modelo4Tanques* which is the nonlinear model of the plant, and the block *MPC Control* which is the Matlab function used for controlling the plant. Everything is already set up, like the 4 changes of reference and the conversion from absolute variables to integral variables.

The only block you have to change is the *MPC Control* block where you should put a Matlab function with your MPC controller for the linearized model. This block has 10 inputs which are the current state of the system (h_1, h_2, h_3, h_4) , the state reference $(h_{1,r}, h_{2,r}, h_{3,r}, h_{4,r})$ and the input reference $(q_{a,r}, q_{b,r})$. The output of the block are the two input to the system (q_a, q_b) . Notice that these variables h_i and q_j are incremental variables, the deviation with respect the operation point. The conversion to the absolute variables is made outside the *MPC Control* block.

- Before running a simulation, the file called *inicio4tanques.m* must be executed. This file initializes the plant model.

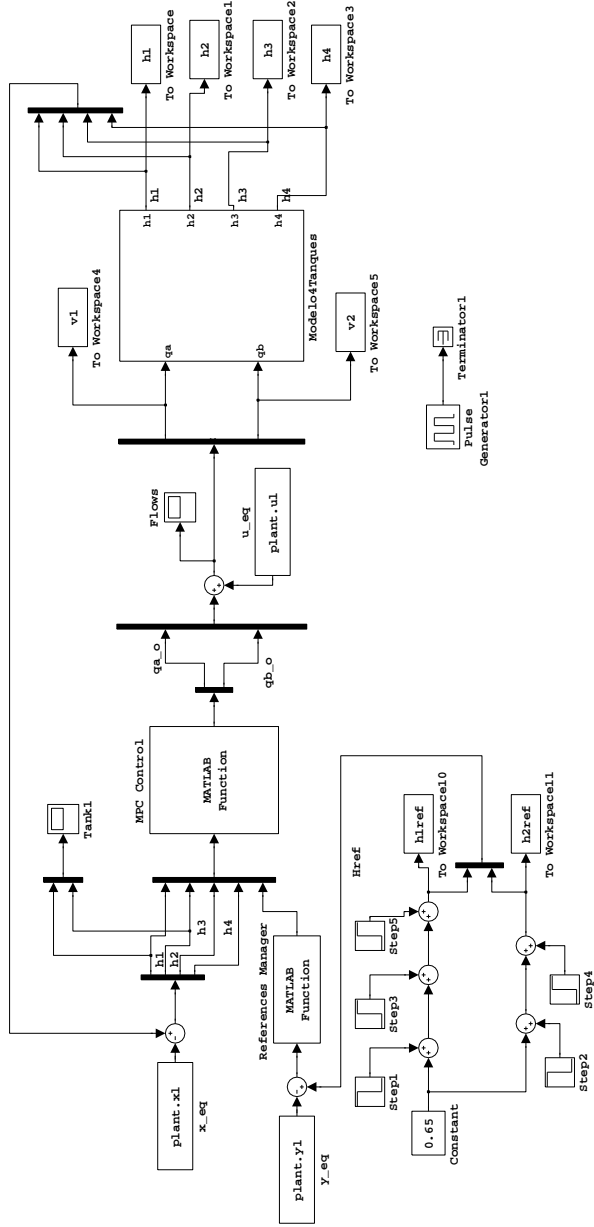


Figure 3: Simulink model.

- At the end of any simulation you can run the file *graficas.m* to plot your results.

With the simulink model, we provide you an example with a centralized MPC controller designed with the parameters mentioned above.